UNIVERSIDAD DE ORIENTE
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ÁREA DE MATEMÁTICA
ASIGNATURA: MATEMATICAS IV
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EJERCICIOS DE ECUACIONES DIFERENCIALES DESARROLLADOS POR SERIES INFINITAS.

1) Determine la solución de y´´ -2xy´ + y = 0

Sustituyendo $y = \sum_{n=0}^{\infty} c_n x^n$ en la ecuación del diferencial nosotros tenemos

$$y^{``} - 2xy^{`} + y = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - 2\sum_{n=1}^{\infty} nc_n x^n + \sum_{n=0}^{\infty} c_n x^n$$

$$= \sum_{k=0}^{\infty} (k+2)(k+1)c_k + 2x^k - 2\sum_{k=1}^{\infty} kc_k x^k + \sum_{k=0}^{\infty} c_n x^k$$

$$= 2c_2 + c_o + \sum_{k=1}^{\infty} [(k+2)(k+1)c_k + 2 - (2k-1)c_k]x^k = 0$$

Así

$$2c_2 + c_o = 0$$

$$(k+2)(k+1)c_k + 2 - (2k-1)c_k = 0$$

y

$$C2 = -\frac{1}{2}c_o$$

$$c_k + 2 = \frac{2k-1}{(k+2)(k+1)}c_{k,}\ k = 1,2,3 \ldots.$$

Escogiendo c_o= 1 y C_1 = 0 nosotros encontramos

$$c_2 = -\frac{1}{2}$$

$$c_3 = c_5 = c_7 = \cdots = 0$$

$$c_4 = -\frac{1}{8}$$

$$c_6 = -\frac{7}{336}$$

Para $C_0 = 0$ y $C_1 = 1$ nosotros encontramos

$$c_2 = c_4 = c_6 = \ldots = 0$$

$$c_3 = \frac{1}{6}$$

$$c_5 = \frac{1}{24}$$

$$c_7 = \frac{1}{112}$$

Y así sucesivamente. Así, dos soluciones son.

$y1 = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{7}{336}x^6 - \ldots$ y $y2 = x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{1}{112}x^7 + \ldots$

2) Determine la solución de y´´- xy´ + 2y = 0

Sustituyendo $y = \sum_{n=0}^{\infty} c_n x^n$ en la ecuación del diferencial nosotros tenemos:

$$y^{``} - xy^{`} + 2y = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - \sum_{n=1}^{\infty} nc_n x^n + 2\sum_{n=0}^{\infty} c_n x^n$$

$$= \sum_{k=0}^{\infty} (k+2)(k+1)c_k + 2x^k - \sum_{k=1}^{\infty} kc_k x^k + 2\sum_{k=0}^{\infty} c_n x^k$$

$$= 2c_2 + 2c_o + \sum_{k=1}^{\infty} [(k+2)(k+1)c_k + 2 - (k-2)c_k]x^k = 0$$

Así

$$2c_2 + 2c_o = 0$$

$$(k+2)(k+1)c_k + 2 - (k-2)c_k = 0$$

Y

$$C2 = -c_o$$

Escogiendo $C_o = 1$ y $C_1 = 0$ nosotros encontramos

$$c_2 = -1$$
$$c_3 = c_5 = c_7 = \cdots = 0$$
$$c_4 = 0$$
$$c_6 = c_8 = c_{10} = \ldots = 0$$

Para $C_0 = 0$ y $C_1 = 1$ nosotros encontramos

$$c_2 = c_4 = c_6 = \ldots = 0$$
$$c_3 = -\frac{1}{6}$$
$$c_5 = -\frac{1}{120}$$

Y así sucesivamente. Así, dos soluciones son.

$y1 = 1 - x^2$ $\qquad$ Y $\qquad$ $y2 = x - \frac{1}{6}x^3 - \frac{1}{120}x^5 - \cdots.$

3) Determinar la solución de y´´ + x²y´ + xy = 0

Sustituyendo $y = \sum_{n=0}^{\infty} c_n x^n$ en la ecuación del diferencial nosotros tenemos:

$$y^{``} + x^2 y^{`} + xy = \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{n=1}^{\infty} nc_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1}$$
$$= \sum_{k=0}^{\infty} (k+2)(k+1)c_k + 2x^k + \sum_{k=2}^{\infty} (k-1)ck - 1x^k + \sum_{k=1}^{\infty} c_{k-1} x^k$$
$$= 2c_2 + (6c_3 + c_0)x + \sum_{k=2}^{\infty} [(k+2)(k+1)\ c_{x+2}\ + kc_{k-1}]x^k = 0$$

Así

$$c2 = 6c_3 + c_o = 0$$
$$(k+2)(k+1)c_{k-2} + kc_{k-1} = 0$$

y

$$C3 = -\frac{1}{6}c_o$$
$$c_k + 2 = \frac{k}{(k+2)(k+1)} c_{k-1},\ k = 2,3,4 \ldots.$$

Escogiendo c_o= 1 y C_1 = 0 nosotros encontramos

$$c_3 = -\frac{1}{6}c_0$$

$$c_4 = c_5 = 0$$

$$c_6 = -\frac{1}{45}$$

Para C_0 = 0 y C_1= 1 nosotros encontramos

$$c_3 = 0$$

$$c_4 = -\frac{1}{6}$$

$$c_5 = c_6 = 0$$

$$c_7 = \frac{5}{252}$$

Y así sucesivamente. Así, dos soluciones son.

$y1 = 1 - \frac{1}{6}x^3 + \frac{1}{45}x^6 - \ldots \quad y \qquad y2 = x - \frac{1}{6}x^4 + \frac{5}{232}x^7$.....

4) $y' + y = 0$

$$\sum_{n=1}^{\infty} nC_n x^{n-1} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$k = n - 1 \;\; k = n$

$$\sum_{k=0}^{\infty} (k+1)C_{k+1}x^k + \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} x^k \left[(k+1)C_{k+1} + C_k\right] = 0 \Rightarrow C_{k+1} = -\frac{C_k}{k+1}$$

$$k = 0 \Rightarrow C_1 = -C_0$$

$$k = 1 \Rightarrow C_2 = \frac{-C_1}{2} = \frac{-(-C_0)}{2} = \frac{C_0}{2}$$

$$k = 2 \Rightarrow C_3 = \frac{-C_2}{3} = \frac{-\left(C_0/2\right)}{3} = \frac{-C_0}{6}$$

$$k = 3 \Rightarrow C_4 = \frac{-C_3}{4} = \frac{-\left(-C_0/6\right)}{4} = \frac{C_0}{24}$$

$$k = 4 \Rightarrow C_5 = \frac{-C_4}{5} = \frac{-\left(C_0/24\right)}{5} = \frac{-C_0}{120}$$

$$y = C_0 - C_0 x + \frac{C_0}{2} x^2 - \frac{C_0}{6} x^3 + \frac{C_0}{24} x^4 - \frac{C_0}{120} x^5$$

$$y = C_0 \left[1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} \right]$$

$$y = C_0 \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = C_0 e^{-x}$$

5) $y' - 2y = 0$

$$\sum_{n=1}^{\infty} n C_n x^{n-1} - 2\sum_{n=0}^{\infty} C_n x^n = 0$$

$$k = n - 1 \quad k = n$$

$$\sum_{k=0}^{\infty} (k+1) C_{k+1} x^k - 2\sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} x^k \left[(k+1) C_{k+1} - 2C_k \right] = 0 \Rightarrow C_{k+1} = \frac{2C_k}{k+1}$$

$$k = 0 \Rightarrow C_1 = 2C_0$$

$$k = 1 \Rightarrow C_2 = \frac{2C_1}{2} = 2C_0$$

$$k = 2 \Rightarrow C_3 = \frac{2C_2}{3} = \frac{2(2C_0)}{3} = \frac{4C_0}{3}$$

$$k = 3 \Rightarrow C_4 = \frac{2C_3}{4} = \frac{2}{4}\left(\frac{4C_0}{3}\right) = \frac{8C_0}{12}$$

$$k = 4 \Rightarrow C_5 = \frac{2C_4}{5} = \frac{2}{5}\left(\frac{8C_0}{12}\right) = \frac{16C_0}{60}$$

$$y = C_0 + 2C_0 x + 2C_0 x^2 + \frac{4C_0}{3}x^3 + \frac{8C_0}{12}x^4 + \frac{16C_0}{60}x^5$$

$$y = C_0\left[1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{8}{12}x^4 + \frac{16}{60}x^5\right] = C_0\sum_{n=0}^{\infty}\frac{(2x)^n}{n!} \Rightarrow y = C_0 e^{2x}$$

6) $y' - x^2 y = 0$

$$\sum_{n=0}^{\infty} nC_n x^{n-1} - x^2\sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2} = 0$$

$$C_1 + C_2 x + \sum_{n=3}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2} = 0$$

$k = n - 3 \quad k = n$

$$\sum_{k=0}^{\infty}(k+3)C_{k+3}x^{k+2} - \sum_{k=0}^{\infty} C_k x^{k+2} = 0$$

$$\sum_{k=0}^{\infty} x^{k+2}\left[(k+3)C_{k+3} - C_k\right] = 0 \Rightarrow C_{k+3} = \frac{C_k}{k+3}$$

$$k = 0 \Rightarrow C_3 = \frac{C_0}{3}$$

$$k = 1 \Rightarrow C_4 = \frac{C_1}{4} = 0$$

$$k = 2 \Rightarrow C_5 = \frac{C_2}{5} = 0$$

$$k = 3 \Rightarrow C_6 = \frac{C_3}{6} = \frac{C_0}{18}$$

$$k = 4 \Rightarrow C_7 = \frac{C_4}{7} = 0$$

$$k = 5 \Rightarrow C_8 = \frac{C_5}{8} = 0$$

$$k = 6 \Rightarrow C_9 = \frac{C_6}{9} = \frac{C_0}{162}$$

$$y = C_0 + \frac{C_0}{3}x^3 + \frac{C_0}{18}x^6 + \frac{C_0}{162}x^9$$

$$y = C_0\left[1 + \frac{x^3}{3} + \frac{x^6}{18} + \frac{x^9}{162}\right]$$

$$y = C_0\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{x^3}{3}\right)^n = C_0 e^{x^3/3}$$

7) $y'' + y = 0$

$$\sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$k = n - 2 \quad k = n$

$$\sum_{k=0}^{\infty} (k+2)(k+1)C_{k+2} x^k - \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} x^k \left[(k+2)(k+1)C_{k+2} + C_k\right] = 0 \Rightarrow C_{k+2} = \frac{-C_k}{(k+2)(k+1)}$$

$$k = 1 \Rightarrow C_3 = \frac{-C_1}{6}$$

$$k = 2 \Rightarrow C_4 = \frac{-C_2}{12} = \frac{-\left(-\frac{C_0}{2}\right)}{12} = \frac{C_0}{24}$$

$$k = 3 \Rightarrow C_5 = \frac{-C_3}{20} = \frac{-\left(-\frac{C_1}{6}\right)}{20} = \frac{C_1}{120}$$

$$k = 4 \Rightarrow C_6 = \frac{-C_4}{30} = \frac{-\left(\frac{C_0}{24}\right)}{30} = \frac{-C_0}{720}$$

$$k = 5 \Rightarrow C_7 = \frac{-C_5}{42} = \frac{-\left(\frac{C_1}{120}\right)}{42} = \frac{-C_1}{5040}$$

$$k = 6 \Rightarrow C_8 = \frac{-C_6}{56} = \frac{-\left(-\frac{C_0}{720}\right)}{56} = \frac{C_0}{40320}$$

$$y = C_0 + C_1 x - \frac{C_0}{2}x^2 - \frac{C_1}{6}x^3 + \frac{C_0}{24}x^4 + \frac{C_1}{120}x^5 - \frac{C_0}{720}x^6 - \frac{C_1}{5040}x^7 + \frac{C_0}{40320}x^8$$

$$y_1 = C_0 - \frac{C_0}{2}x^2 + \frac{C_0}{24}x^4 - \frac{C_0}{720}x^6 + \frac{C_0}{40320}x^8$$

$$y_1 = C_0\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}\right] = C_0 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = C_0 \cos x$$

$$y_2 = C_1 x - \frac{C_1}{6}x^3 + \frac{C_1}{120}x^5 - \frac{C_1}{5040}x^7$$

$$y_2 = C_1\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right] = C_1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = C_1 senx$$

$$y = y_1 + y_2 = C_0 \cos x + C_1 senx$$

8) Resuelva la siguiente ecuación diferencial por medio de series:

$$\dot{y} + y = 0$$

- Serie de potencia:

$$y = \sum_{n=0}^{\infty} C_n X^n$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} n C_n X^{n-1} = \sum_{n=1}^{\infty} n C_n X^{n-1}$$

$$\dot{y} + y = \sum_{n=1}^{\infty} n C_n X^{n-1} + \sum_{n=0}^{\infty} C_n X^n$$

De tal manera que para la primera serie, tenemos: $k = n - 1 \quad \Rightarrow \quad n = k + 1$

Y para la segunda serie: $k = n$

$$\dot{y} + y = \sum_{k=0}^{\infty} (k+1) C_{k+1} X^k + \sum_{k=0}^{\infty} C_k X^k$$

$$\dot{y} + y = \sum_{k=0}^{\infty} [(k+1)(C_{k+1}) + C_k] X^k = 0$$

$$(k+1)(C_{k+1}) + C_k = 0$$

$$C_{k+1} = -\frac{C_k}{k+1}$$

k=0 $\quad C_1 = -C_0$

k=1 $\quad C_2 = -\frac{1}{2}C_1 = \frac{1}{2!}C_0$

k=2 $\quad C_3 = -\frac{1}{3}C_2 = -\frac{1}{3 \cdot 2!}C_0 = -\frac{1}{3!}C_0$

k=3 $\quad C_4 = -\frac{1}{4}C_3 = -\frac{1}{4 \cdot 3!}C_1 = \frac{1}{4!}C_0$

k=4 $\quad C_5 = -\frac{C_4}{5} = -\frac{1}{5 \cdot 4!}C_0 = -\frac{1}{5!}C_0$

k=5 $\quad C_6 = -\frac{1}{6}C_5 = \frac{1}{6 \cdot 5!}C_0 = \frac{1}{6!}C_0$

k=6 $\quad C_7 = -\frac{1}{7}C_6 = -\frac{1}{7 \cdot 6!}C_0 = -\frac{1}{7!}C_0$

$$y = \sum_{n=0}^{\infty} C_n X^n$$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7 \cdots$$

$$y = C_0 - C_0 x + \frac{1}{2!} x^3 C_0 - \frac{1}{3!} C_0 x^3 + \frac{1}{4!} C_0 x^4 - \frac{1}{5!} C_0 x^5 + \frac{1}{6!} C_0 x^6 - \frac{1}{7!} C_0 x^2 + \cdots$$

$$y = C_0 \left[1 - x + \frac{1}{2!} x^2 - \frac{1}{3!} x^3 + \frac{1}{4!} x^4 - \frac{1}{5!} x^5 + \frac{1}{6!} x^6 - \frac{1}{7!} x^7 + \cdots \right]$$

$$y = C_0 \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!}$$

9) Resuelva la siguiente ecuación diferencial por medio de series:

$$\dot{y} - 2y = 0$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} n C_n x^{n-1} = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$\dot{y} - 2y = \sum_{n=1}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} 2 C_n x^n$$

$$\dot{y} - 2y = C_1 + \sum_{n=2}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} 2 C_n x^n$$

$$\dot{y} - 2y = C_1 + \sum_{n=2}^{\infty} n C_n x^{n-1} - 2C_0 - \sum_{n=1}^{\infty} 2 C_n x^n$$

$$\dot{y} - 2y = C_1 + \sum_{k=1}^{\infty}(k+1)C_{k+1}x^k - 2C_0 - \sum_{k=1}^{\infty} 2\,C_n x^k$$

$$\dot{y} - 2y = C_1 - 2C_0 + \sum_{k=1}^{\infty}[(k+1)C_{k+1} - 2C_k]x^k = 0$$

$$C_1 - 2C_0 = 0$$

$$C_1 = 2C_0$$

$$(k+1)C_{k+1} - 2C_k = 0$$

$$C_{k+1} = \frac{2C_k}{k+1}$$

k= 1 $\quad C_2 = \frac{2}{2}C_1 = C_1 = 2C_0$

k=2 $\quad C_3 = \frac{2}{3}C_2 = \frac{2}{3}2C_0 = \frac{4}{3!}C_0$

k=3 $\quad C_4 = \frac{2}{4}C_3 = \frac{8}{4!}C_0$

k=4 $\quad C_5 = \frac{2}{5}C_4 = \frac{16}{5!}C_0$

k=5 $$C_6 = \frac{2}{6}C_5 = \frac{2 \cdot 16}{6 \cdot 5!}C_0 = \frac{32}{6!}C_0$$

$$y = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 + C_6x^6 + \cdots$$

$$y = C_0 + 2C_0x + 2C_0x^2 + \frac{4}{3!}C_0x^3 + \frac{8}{4!}C_0x^4 + \frac{16}{5!}C_0x^5 + \frac{32}{6!}C_0x^6 + \cdots$$

$$y = C_0\left[1 + 2x + 2x^2 + \frac{4}{3!}x^3 + \frac{8}{4!}x^4 + \frac{16}{5!}x^5 + \frac{32}{6!}x^6 + \cdots\right]$$

10) Resuelva la siguiente ecuación diferencial lineal:

$$\dot{y} - x^2y = 0$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} nC_nx^{n-1} = \sum_{n=1}^{\infty} nC_nx^{n-1}$$

$$\dot{y} - x^2y = \sum_{n=1}^{\infty} nC_nx^{n-1} - \sum_{n=0}^{\infty} C_nx^{n+2}$$

$$\dot{y} - x^2 y = C_1 + \sum_{n=2}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + \sum_{n=2}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + 2xC_2 + \sum_{n=3}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + 2xC_2 + \sum_{k=2}^{\infty} (k+1)C_{k+1} x^k - \sum_{k=2}^{\infty} C_{k-2} x^k$$

$$\dot{y} - x^2 y = C_1 + 2xC_2 + \sum_{k=2}^{\infty} [(k+1)C_{k+1} - C_{k-2}] x^k$$

$$(k+1)C_{k+1} - C_{k-2} = 0$$

$$C_{k+1} = \frac{C_{k-2}}{k+1}$$

$$C_1 = 0$$

$$2xC_2 = 0 \quad \Rightarrow \quad C_2 = 0$$

k=2 $$C_3 = \frac{C_0}{3} = \frac{1}{3!}C_0$$

k=3 $$C_4 = \frac{1}{4}C_1 = \frac{1}{4}(0) = 0$$

k=4 $$C_5 = \frac{1}{5}C_2 = \frac{1}{5}(0) = 0$$

k=5 $$C_6 = \frac{1}{6}C_3 = \frac{1}{6}\frac{1}{3!}C_0 = \frac{1}{6 \cdot 3!}C_0$$

k=6 $$C_7 = \frac{1}{7}C_4 = \frac{1}{7}(o) = 0$$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7 \cdots$$

$$y = C_0 + 0 + 0 + \frac{1}{3!}C_0 x^3 + 0 + 0 + \frac{1}{6 \cdot 3!}C_0 x^6 + 0 + \cdots$$

$$y = C_0\left[1 + \frac{1}{3!}x^3 + \frac{1}{6 \cdot 3!}x^6\right]$$

11. Resolver la ecuación diferencial $(x-1)y''+y'$

Conocemos que: $y=\sum_{n=0}^{\infty} c_n x^n$ sustituyendo esta expresión en la ecuación diferencial a resolver tenemos:

$$(x-1)y''+y' = \underbrace{\sum_{n=2}^{\infty} n(n-1)c_n x^{n-1}}_{k=n-1} - \underbrace{\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}}_{k=n-2} + \underbrace{\sum_{n=1}^{\infty} nc_n x^{n-1}}_{k=n-1}$$

$$= \sum_{k=1}^{\infty}(k+1)kc_{k+1}x^k - \sum_{k=0}^{\infty}(k+2)(k+1)c_{k+2}x^k + \sum_{k=0}^{\infty}(k+1)c_{k+1}x^k$$

$$= -2c_2 + c_1 + \sum_{k=1}^{\infty}[(k+1)kc_{k+1} - (k+2)(k+1)c_{k+2} + (k+1)c_{k+1}]x^k = 0.$$

Entonces:

$$-2c_2 + c_1 = 0$$

$$(k+1)^2 c_{k+1} - (k+2)(k+1)c_{k+2} = 0$$

Y,

$$c_2 = \frac{1}{2}c_1$$

$$c_{k+2} = \frac{k+1}{k+2}c_{k+1}$$

Luego, haciendo $C_0 = 1$ y , $C_1 = 0$ obtenemos

$$C_2 = C_3 = C_4 = 0$$

Y Haciendo $C_0 = 0$ y , $C_1 = 1$, obtenemos:

$$C_2 = \frac{1}{2}, C_3 = \frac{1}{3}, C_4 = \frac{1}{4}$$

Entonces tenemos las siguientes respuestas:

$$y_1 = 1, y_2 = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$$